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## Abstract

We study the neutral-current neutrino scattering for four nuclei in the iron region. We evaluate the cross sections for the relevant temperatures during the supernova core collapse and derive Gamow-Teller distributions from large-scale shell-model calculations. We show that the thermal population of the excited states significantly enhances the cross sections at low neutrino energies. Calculations of the outgoing neutrino spectra indicate the prospect of neutrino upscattering at finite temperatures. Both results are particularly notable in even-even nuclei.

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Neutrino scattering plays a fundamental role in the evolution of the core collapse of a massive star towards a supernova explosion. Due to the high densities in the star's core, both the transport of electromagnetic radiation and electronic heat conduction are very slow compared to the short time scale of the collapse [1]. Thus, until the core reaches densities of  $\rho \approx 4 \times 10^{11} \text{ gcm}^{-3}$ , almost all of the energy of the collapse is transported by the neutrinos that move out of the star essentially without resistance. At higher densities the neutrinos can become trapped inside the core due to elastic scattering on nucleons and nuclei. Subsequently neutrinos lose their energy (downscatter) by inelastic neutrino-electron scattering. This process rapidly leads to an equilibrium between neutrinos and matter [1]. The thermalization gradually builds a Fermi distribution of neutrinos and helps to maintain a larger lepton fraction inside the core.

One might naively expect that this straightforward mechanism helps to attain a larger homologous core that leads to a stronger shock wave and less overlying iron for the shock wave to photodisintegrate. On the other hand, low-energy neutrinos have a longer mean free path and can therefore diffuse more easily out of the core. The net effect of the neutrino downscattering depends strongly on the initial conditions of the star and relies on detailed neutrino transport calculations that include all relevant neutrino reactions. Haxton pointed out that neutral-current neutrino scattering on nuclei involving the excitation of the giant resonances can lead to significant neutrino cross sections and should also be added in supernova simulations [2]. This suggestion was incorporated into a collapse simulation performed by Bruenn and Haxton [3]. They included inelastic neutrino-nucleus scattering, representing the effects of heavy nuclei solely by  $^{56}\text{Fe}$ . The relevant cross sections were calculated on the basis of a strongly truncated shell model for the allowed transitions and the Goldhaber-Teller model for forbidden transitions. These calculations demonstrated that in later stages of the collapse inelastic neutrino-nucleus scattering can compete with inelastic scattering off electrons.

As mentioned above, neutrino interaction with matter is particularly important for low neutrino energies  $E_\nu$ . In this energy range, the neutrino-nucleus rates are found to be smaller

than the  $\nu + e^-$  rates by  $\sim 1 - 2$  orders of magnitude for  $E_\nu \leq 5$  MeV and densities in the range  $10^{10} - 10^{12}$  g/cm<sup>3</sup>; however, we note that the inelastic neutrino-nucleus cross section, obtained for the  $^{56}\text{Fe}$  ground state and used in [3], underestimates the effects of inelastic neutrino-nucleus scattering in a supernova environment for two reasons. First, Gamow-Teller (GT) transitions, which strongly dominate the inelastic cross sections at low neutrino energies, have to overcome the gap between the  $J = 0^+$  ground state (for even-even nuclei like  $^{56}\text{Fe}$ ) and the first excited  $J = 1^+$  state, which resides at around 3 MeV in  $^{56}\text{Fe}$  (a typical excitation energy for this state in even-even nuclei in the iron group). This introduces a threshold for inelastic neutrino scattering on the ground state of even-even nuclei and strongly suppresses the cross sections for low-energy neutrinos. This strong threshold effect is absent for odd- $A$  and odd-odd nuclei, where the ground state has  $J > 0$  and is usually connected to low-lying states by sizable GT transitions. Second, inelastic neutrino-nucleus scattering occurs at finite temperature ( $T \gtrsim 0.8$  MeV). Hence excited states in the nucleus can be thermally populated. This effectively removes the threshold effect in even-even nuclei since the increasing level density with rising temperature allows for many GT transitions. Interestingly, this includes transitions to nuclear states at lower excitation energies. Such transitions correspond to upscattering of neutrinos, as their energy in the final state is larger than in the initial state.

The importance of finite-temperature effects in inelastic neutrino-nucleus scattering was noted by Fuller and Meyer [4] who studied these effects for a few even-even nuclei (including  $^{56}\text{Fe}$ ) on the basis of the independent particle model; however, this model is not well suited for a quantitative description at collapse temperatures ( $\sim 1$  MeV) as it sets the threshold energy for GT transitions effectively by the spin-orbit splitting ( $\sim 7$  MeV for  $f_{7/2} \rightarrow f_{5/2}$  in  $^{56}\text{Fe}$ ).

The description of inelastic neutrino-nucleus scattering at finite temperatures and low neutrino energies requires a model which reproduces both the spectroscopy and the GT strength distribution of the nucleus sufficiently well. For Fe-group nuclei it was recently demonstrated that modern shell model diagonalization calculations using an appropriate

residual interaction are capable of this task [5]. Based on the shell model results of [5,6] we will, in the following, calculate inelastic neutrino cross sections at finite temperatures for the even-even nucleus  $^{56}\text{Fe}$ , the odd- $A$  nuclei  $^{59}\text{Fe}$  and  $^{59}\text{Co}$  and the odd-odd nucleus  $^{56}\text{Co}$ . In previous work we have studied finite-temperature effects on charged-current neutrino reactions, again based on shell model GT distributions [7].

An explicit calculation of the cross section at finite temperature ( $T \gtrsim 1$ ) MeV includes too many states to derive the GT strength distribution for each individual state and is hence unfeasible. We therefore use the following strategy. We split the cross section into parts describing neutrino down-scattering ( $E'_\nu \geq E_\nu$ ) and upscattering ( $E'_\nu \leq E_\nu$ ), where  $E'_\nu, E_\nu$  are the neutrino energies in the final and initial states, respectively. For the downscattering part we apply the Brink hypothesis [11] which states that for a given excited level the GT distribution built on this state,  $S_i(E)$ , is the same as for the ground state ( $S_0(E)$ ), but shifted by the excitation energy  $E_i$ , i.e.,  $S_i(E) = S_0(E - E_i)$ . The Brink hypothesis was proven valid for the GT resonant states; however, it can fail for specific low-lying transitions. We consider important low-lying transitions with large phase space in our second cross section term. Here we include excited states that are connected by GT transitions to the ground state or lowest excited states of the nucleus. These contributions are determined by ‘inversion’ of the shell-model GT distributions of the low-lying states. Then, assuming that the inelastic neutrino-nucleus scattering cross section at low energies ( $E_\nu \lesssim 15$  MeV) is mainly given by GT transitions, our neutral-current neutrino cross section reads:

$$\sigma_\nu(E_\nu) = \frac{G_F^2 \cos^2 \theta_C}{\pi} \left[ \sum_f E_{\nu',0f}^2 B_{0f}(\text{GT}_0) + \sum_{if} E_{\nu',if}^2 B_{if}^{\text{back}}(\text{GT}_0) \frac{G_i}{G} \right] \quad (1)$$

where  $G_F$  is the Fermi constant,  $\theta_C$  the Cabibbo angle, and  $E_{\nu',if}$  is the energy of the scattered neutrino,  $E_{\nu',if} = E_\nu + (E_i - E_f) = E_\nu - q_{0,if}$ , and  $E_i, E_f$  are the initial and final nuclear energies. The first term arises from the Brink’s hypothesis. Under this assumption the nuclear transitions are independent of the initial state and, consequently, the cross section becomes independent of the temperature. For high temperatures, many states will contribute in a way that variations in the low-lying transitions tend to cancel and Brink’s

hypothesis becomes a valid assumption (examples supporting this reasoning can be found in Ref. [6]). The second term accounts for the backresonances contribution, where the sum runs over both initial (i) and final states (f). The former have a thermal weight of  $G_i = (2J_i + 1) \exp(-E_i/kT)$ , where  $J_i$  is the angular momentum, and  $G = \sum_i G_i$  is the nuclear partition function.

All the information about the nuclear structure is comprised in the  $B_{if}(GT_0)$  coefficients, that define the GT strength for the Gamow-Teller operator  $\vec{\sigma}t_0$  between an initial and final state. These were derived from large-scale shell model calculations in the complete  $pf$ -shell and taking a slightly modified version of the KB3 residual interaction, that corrects for the small overbinding at  $N = 28$  shell closure found in the original KB3 force [5].

In Fig. 1 we show the  $GT_0$  strength built on the ground-state of the four selected nuclei (with isospin  $I$ ). One can see that the  $\Delta I = 1$  component has a much lower total strength than the  $\Delta I = 0$  component. This reduction is simply due to the geometrical factor that relates the  $GT_0$  matrix element to its reduced matrix element and increases with larger neutron excess. The major contribution of the  $\Delta I = 0$   $GT_0$  strength distribution is concentrated in a resonant region around  $E = 10$  MeV for all four nuclei. (We will refer to this concentration of strength as the GT resonance.) Hence the position of the  $GT_0$  resonance does not depend on the pairing structure of the nuclear ground state (see also [12]). The same behavior was already noticed for the centroids of the charge-changing GT distributions [6]. Remembering that the Brink hypothesis is quite accurate for the GT resonances one can already conclude here that inelastic neutrino excitation of the dominant  $GT_0$  transitions requires  $E_\nu \gtrsim 10$  MeV. Furthermore, a thermal excitation of these states, considered in the upscattering component, is strongly suppressed at temperatures of order 1 MeV. Hence, the neutrino cross section for  $E_\nu \leq 10$  MeV will be strongly influenced by the rather weak low-energy tail of the  $GT_0$  distribution. In contrast to the resonance peak, this tail is quite dependent on the pairing structure. For odd- $A$  and odd-odd nuclei (with  $J \geq 0$ ) modest  $GT_0$  transitions are possible to states at very small excitation energies, effectively avoiding a threshold for inelastic scattering. This is clearly different for the even-even nucleus  $^{56}\text{Fe}$

where  $GT_0 = 0$  for  $E \lesssim 3$  MeV. The  $\Delta I = 1$  component of the  $GT_0$  distribution resides at such high excitation energies (centred around 2 MeV higher than the centroids of the  $\Delta I = 0$  part for the nuclei studied here), that it cannot be reached by low-energy neutrino scattering in the upward component and is strongly thermally suppressed in the downward component of the cross section.

Fig. 2 shows the inelastic neutrino cross sections for the four nuclei, calculated from Eq. 1. We performed the calculations for the ground state  $GT_0$  distribution, which simulates the  $T = 0$  case, and at 3 different temperatures ( $T = 0.86$  MeV,  $1.29$  MeV,  $1.72$  MeV), where  $T = 0.86$  MeV corresponds to the condition of a presupernova model for a  $15M_\odot$  star ( $\rho \sim 10^{10}$  g/cm<sup>3</sup> [9]). The two other temperatures relate approximately to neutrino trapping ( $T = 1.29$  MeV) and thermalization ( $T = 1.72$  MeV). Our calculations do not include neutrino blocking in the final state. For all nuclei we obtain an enhancement of the cross sections at finite temperature due to the thermal population of the backresonances, i.e., the upwards component. Due to the energy gap in the  $GT_0$  distribution this is particularly pronounced for  $^{56}\text{Fe}$  below  $E_\nu \approx 10$  MeV. For  $T = 0$  the cross section drops rapidly to zero as it approaches the reaction threshold ( $\approx 3$  MeV for  $^{56}\text{Fe}$ ). At finite temperature, the gap is then filled by thermal population of states with  $GT_0$  transitions to the ground and lowest excited states. For the odd-A and odd-odd nuclei there is no finite threshold at zero temperature and the increase of the low-energy cross section with temperature is much less dramatic than for even-even nuclei. Finite temperature effects are unimportant for  $E_\nu \geq 10$  MeV where inelastic excitation of the  $GT_0$  resonance becomes possible and dominates the cross sections.

We further note that the cross sections for  $^{59}\text{Fe}$ ,  $^{59}\text{Co}$  and  $^{56}\text{Co}$  are quite similar at finite temperatures indicating the prospect that inelastic neutrino-nucleus scattering, at least for odd-A and odd-odd nuclei can be well represented by an ‘average nucleus’ in collapse simulations. We will investigate this point more fully in future research that will also include forbidden transitions. The  $^{56}\text{Fe}$  cross section is smaller than that of the other nuclei at all temperatures considered here, although this effect is significantly diminished with increasing

temperature.

As Bruenn and Haxton compared the inelastic neutrino-electron and neutrino- $^{56}\text{Fe}$  rates, where the latter was evaluated using shell-model ground-state  $\text{GT}_0$  distributions at low neutrino energies, it is interesting to compare the increase of the neutrino-nucleus rate quantitatively. Collapse simulations usually bin the rates in energy intervals of a few MeV ( $\sim 5$  MeV). Thus, we averaged the various inelastic cross sections between 0 and 5 MeV and observed an increase of the  $^{56}\text{Fe}(\nu, \nu')$  cross section by about a factor 30 as the temperature is increased from  $T = 0$  to  $T = 1.29$  MeV. For  $T = 0$  the averaged cross section for the other nuclei is nearly 200 times larger than the one for  $^{56}\text{Fe}$ . At  $T = 1.29$  MeV the cross section for  $^{59}\text{Fe}$ ,  $^{59}\text{Co}$  and  $^{56}\text{Co}$  are also increased by about 30% due to upscattering contributions. This implies that the inelastic neutrino-nucleus scattering rate is larger at low energies than assumed in [3] by more than an order of magnitude.

During the collapse, inelastic scattering off electrons and nuclei is most important in the thermalization phase with densities between  $\rho \sim 10^{11} - 10^{12}$  g/cm $^3$  and temperatures  $T \sim 1 - 1.75$  MeV. Under such conditions neutrinos are produced mainly by electron capture on free protons and hence have energies of order 8 – 15 MeV [3]. If we compare the various inelastic neutrino cross sections, for example at  $T = 1.29$  MeV, with the  $^{56}\text{Fe}$   $T = 0$  cross section, as used in [3], we find an increase of about a factor 5-6 for  $E_\nu = 10$  MeV for the two cobalt isotopes; at  $E_\nu = 15$  MeV this increase compared to the  $T = 0$   $^{56}\text{Fe}$  cross section is reduced to about 30%. Hence, we conclude that nuclear structure and temperature effects might still be relevant for 10 MeV neutrinos, but can be neglected for  $E_\nu \gtrsim 15$  MeV.

In our model, the temperature-related increase of the cross section is due to the upscattering contributions when the neutrino gains energy by nuclear deexcitation. We note that the previously discussed downscattering contributions in inelastic neutrino scattering increase the entropy of the collapse environment as they result in nuclear excitation. In contrast, the upscattering contributions reduce the entropy. The relative importance of the down- and upscattering components can also be read directly from the neutrino energy distributions resulting from the inelastic scattering process. Such double-differential dis-

tributions are also of interest for the neutrino transport simulations [10]. We present two representative neutrino spectra for initial  $E_\nu = 7.5$  MeV and 20 MeV neutrinos in Figs. 3 and 4, respectively. The spectra are normalized to unity. Neutrinos with  $E_\nu = 7.5$  MeV are typical for the early collapse stage, while neutrino energies of about 20 MeV are encountered during the thermalization phase at densities around  $10^{12}$  g/cm<sup>3</sup>. For  $E_\nu = 7.5$  MeV neutrinos, the upscattering contributions noticeably increase the cross sections for  $^{56}\text{Fe}$ . Related to this, we also find a significant portion of the spectrum at energies  $E'_\nu \geq 10$  MeV. With increasing temperature, the thermal population of the excited states increases and so do the upscattering contributions to cross sections and spectra. The sizable bump in the spectrum at around  $E'_\nu \sim 17$  MeV corresponds to thermal excitations of the  $\text{GT}_0$  resonance. Except for minor high-energy wings with  $E'_\nu \geq 7.5$  MeV, reflecting upscattering, finite temperature effects are rather unimportant for the normalized neutrino spectra in inelastic scattering off the other 3 nuclei. The spectra for these nuclei, in particular for  $^{56,59}\text{Co}$ , are dominated by peaks around the incoming neutrino energy ( $E_\nu = 7.5$  MeV) implying that excitation and deexcitation of low-lying excited states dominate the cross sections.

We note that the  $\text{GT}_0$  resonance cannot be reached in the downscattering direction for  $E_\nu = 7.5$  MeV neutrinos. This is different for  $E_\nu = 20$  MeV neutrinos. Consequently the cross section and spectra are entirely dominated by the downscattering contributions for all four nuclei, as already reasoned above (Fig. 4). The noticeable peak in the spectra around  $E'_\nu = 10$  MeV correspond to the inelastic excitation of the  $\text{GT}_0$  resonance. In a quantitative calculation of the cross section in this energy regime forbidden transitions have to be included [12]; however, they do not change our conclusion that finite temperature effects can be neglected.

In summary, we have shown that finite temperature enhances the neutrino inelastic scattering cross sections for low-energy neutrinos. The increase is most significant for even-even nuclei. Our results suggest that inelastic neutrino-nucleus scattering rates might be comparable to the inelastic neutrino scattering off electrons at low neutrino energies. The increase in cross section is related to deexcitation of thermally populated states and thus



reduces the entropy and scatters the neutrinos up in energy. We also find that thermal effects will only influence the cross section for neutrino energies which do not allow an excitation of the  $GT_0$  resonance. As these resonances reside around  $E = 10$  MeV in  $pf$ -shell nuclei, thermal effects are restricted to neutrinos with  $E_\nu \leq 10$  MeV.

Our calculation also indicates rather mild variations of the inelastic neutrino cross section for different nuclei at collapse temperatures. This implies the possibility to derive a double-differential cross section for an ‘average nucleus’ for implementation in collapse simulations, at least for odd-A and odd-odd nuclei. Such work is in progress.

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# FIGURES

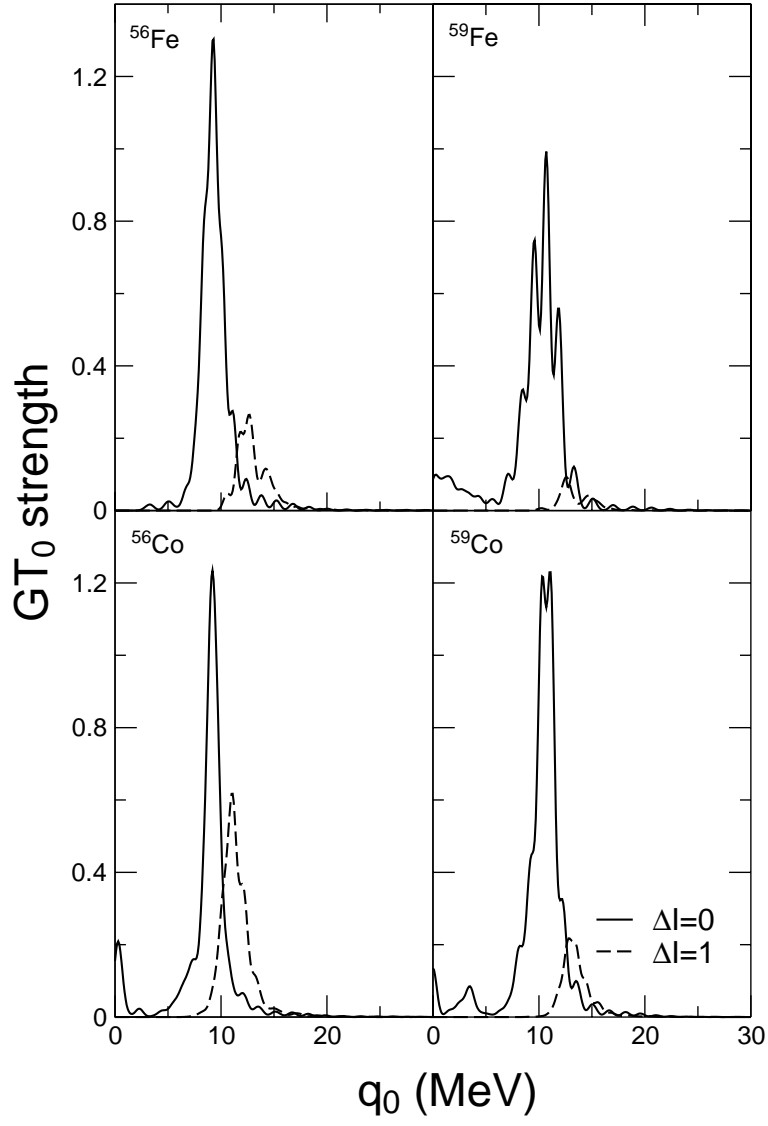


FIG. 1. Distribution of the GT<sub>0</sub> strength built on the ground-state for the four nuclei selected for discussion throughout this letter. The strength is split into the two isospin components:  $\Delta I = 0$  (full line) and  $\Delta I = 1$  (dashed line). The energy scale  $q_0 = E_f - E_i$  refers to the nuclear excitation energy which is equivalent to the neutrino energy transfer.

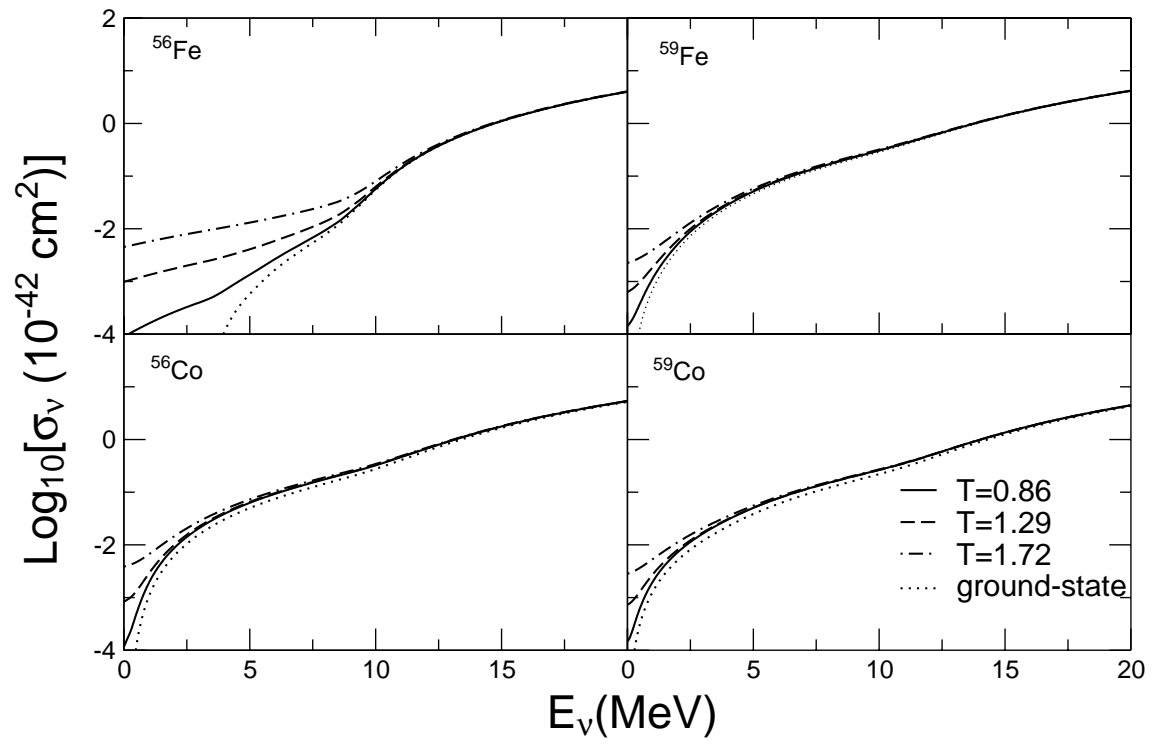


FIG. 2. Neutrino cross sections from neutrino scattering on nuclei at finite temperature. The temperatures are given in MeV. The finite temperature results, derived from Eq. 1, are compared to the one derived solely from the nuclear ground-state.

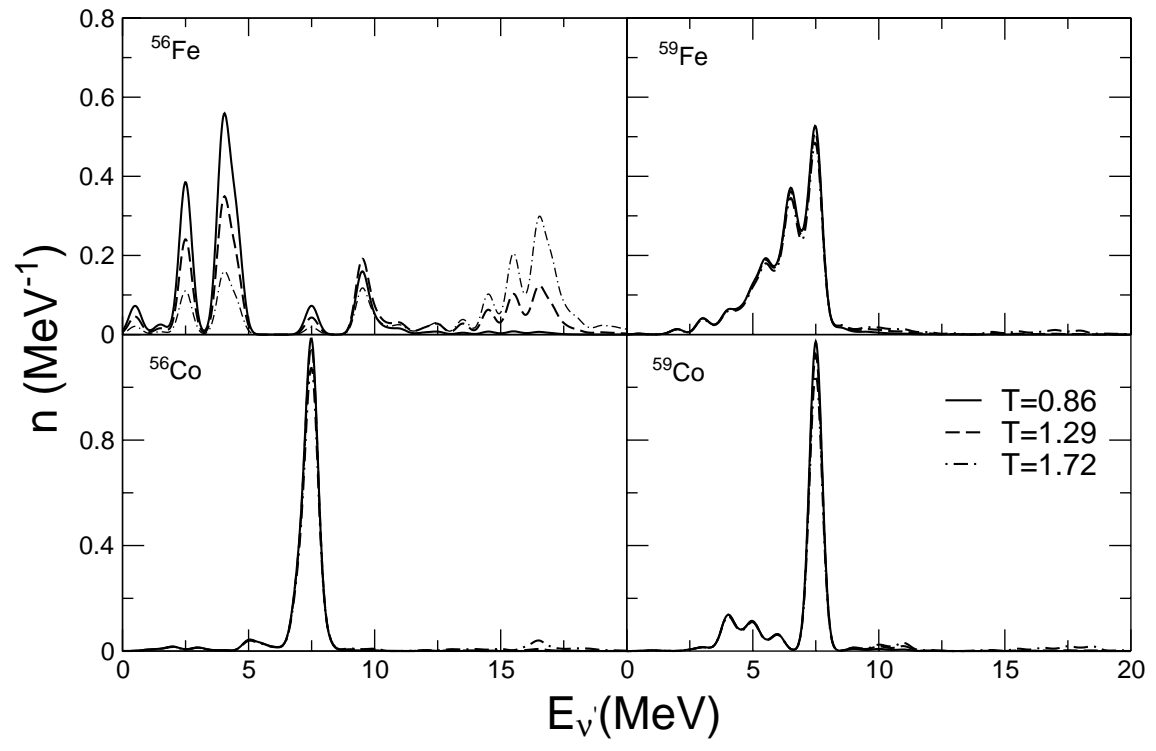


FIG. 3. Normalized neutrino spectra for inelastic scattering of  $E_\nu = 7.5$  MeV neutrinos on nuclei at finite temperature. The temperatures are given in MeV.

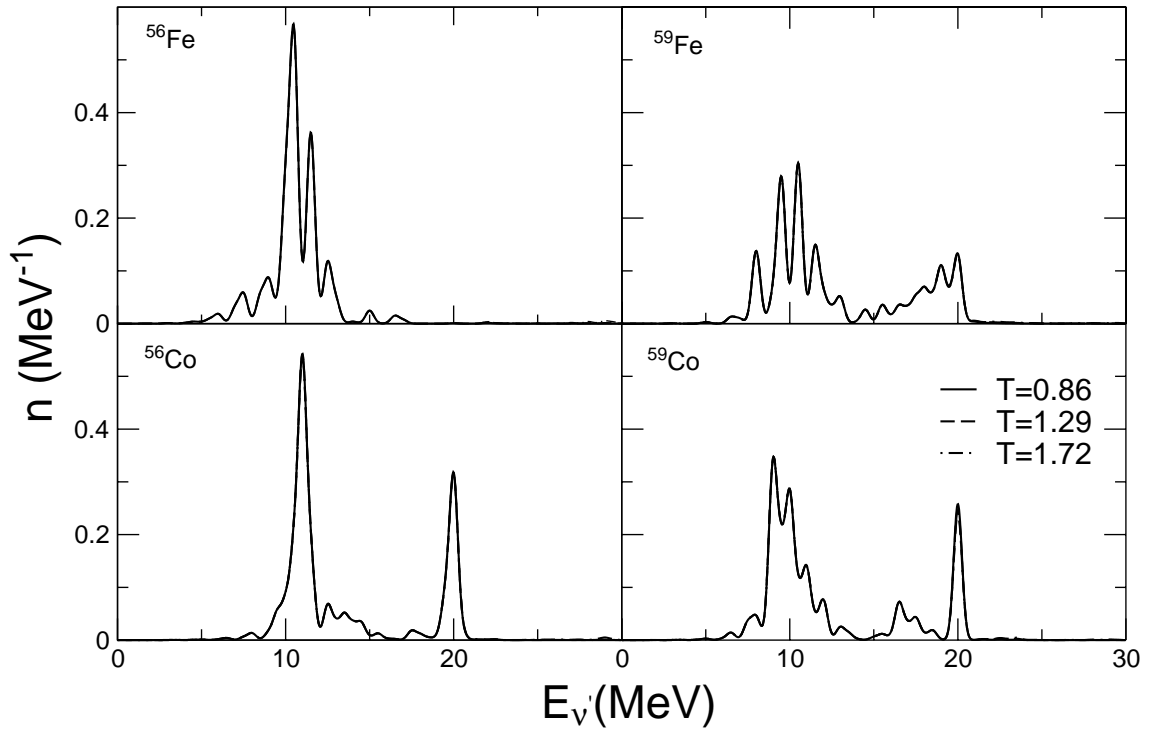


FIG. 4. Normalized neutrino spectra for inelastic scattering of  $E_\nu = 20$  MeV neutrinos on nuclei at finite temperature. The temperatures are given in MeV. Note that the three temperature curves completely overlap, as the spectra are dominated by the (temperature-independent) downscattering components.